PARAMETRIC EXCITATION OF WAVES IN AN ISOTHERMAL PLASMA

B. I. Sturman

UDC 533.9

The linear theory of the parametric excitation of waves in a nonisothermal plasma by a high-frequency field of frequency ω_0 ($\omega_0 \geqslant \omega_{pe}$, ω_{pe} is the plasma electron frequency) predicts the existence of two regions in k-space with different excitation thresholds. The decay instability [1] displays the lower threshold (and a higher increment).

In the present report it is shown that a simplified dynamic description of the plasma, based on an average over the fast time ω_{pe}^{-1}, preserves all the kinetic effects. The dispersion equation obtained on its basis is applied to the study of the instability of an isothermal plasma. It is shown that the two thresholds can be comparable when $T_e \sim T_i$; the increments in the decay instability in this region are determined.

Let us consider the excitation of long-wave ($kr_d \ll 1$, r_d is the Debye radius) plasma oscillations by a uniform field $\mathbf{E} = \mathbf{E}_0 \sin \omega_0 t$ with a frequency ω_0 close to the plasma frequency ω_{pe}. For a description of the Langmuir oscillations it is sufficient to confine oneself to the hydrodynamic equations averaged over the fast time ω_{pe}^{-1} [2]. In an approximation linear with respect to the increasing disturbances we have

$$\frac{\partial \psi_k}{\partial t} + (i\omega_k + \gamma)\psi_k = -\frac{\omega_{pe}}{2n_0k^2}(\mathbf{k}\mathbf{E}_0)\,\delta n_k e^{-i(\omega_0-\omega_{pe})t} \tag{1}$$

Here ψ_k is the Fourier component of the slow amplitude of the high-frequency part of the electrostatic potential φ_e

$$\varphi_e = {}^1/_2(\psi e^{-i\omega_{pe}t} + \psi^* e^{i\omega_{pe}t}) \tag{2}$$

δn is the low-frequency variation in the electron (and ion) density relative to the equilibrium density n_0 ($n_0 \gg \delta n$), $\omega_k = 3/2\,\omega_{pe}(kr_d)^2$, and γ is the decrement in the attenuation of the Langmuir waves.

Slow movements of a plasma with an arbitrary ratio of electron and ion temperatures must be described kinetically. We can account for the effect of the Langmuir oscillations on these motions as the effect on the electrons of a "high-frequency" force with the potential

$$U = \frac{e^2 \mathbf{E}_0}{8m\omega_{pe}^2}[e^{i(\omega_0-\omega_{pe})t}\nabla\psi + e^{-i(\omega_0-\omega_{pe})t}\nabla\psi^*] \tag{3}$$

For the distribution functions $f^{e,i}$ we have [3]

$$\frac{\partial f^i}{\partial t} + \mathbf{v}_i\nabla f^i - \frac{e}{M}\nabla\varphi\frac{\partial f_0^i}{\partial \mathbf{v}_i} = 0, \quad \frac{\partial f^e}{\partial t} + \mathbf{v}_e\nabla f^e - \frac{1}{m}\nabla(U - e\varphi)\frac{\partial f_0^e}{\partial \mathbf{v}_e} = 0 \tag{4}$$

where φ is the slow part of the electrostatic potential and $f_0^{e,i}$ are the Maxwell distributions for the electrons and ions

$$\delta n = \int (f^{e,i} - f_0^{e,i})\,d\mathbf{v} \tag{5}$$

Let us set

$$f^{e,i} - f_0^{e,i} = \delta f_{\omega k}^{e,i} e^{-i\omega t + i\mathbf{kr}} + \delta f_{\omega k}^{*e,i} e^{i\omega t - i\mathbf{kr}}, \quad \varphi = \varphi_{\omega k} e^{-i\omega t + i\mathbf{kr}} + \varphi_{\omega k}^* e^{i\omega t - i\mathbf{kr}}$$
$$\psi = \psi_+ e^{-i(\omega+\omega_0-\omega_{pe})t + i\mathbf{kr}} + \psi_- e^{i(\omega^*-\omega_0+\omega_{pe})t - i\mathbf{kr}} \tag{6}$$

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 2, pp. 157-159, March-April, 1974. Original article submitted September 20, 1972.

© 1975 Plenum Publishing Corporation, 227 West 17th Street, New York, N.Y. 10011. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, microfilming, recording or otherwise, without written permission of the publisher. A copy of this article is available from the publisher for $15.00.

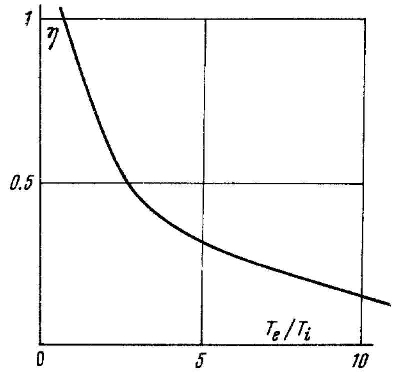

Fig. 1

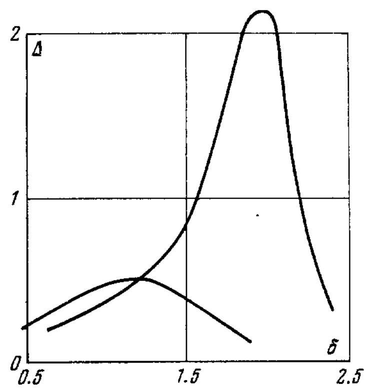

Fig. 2

We obtain the following dispersion equation for the complex frequency ω:

$$[(\omega_{pe}+\omega_k-\omega_0)^2-(\omega+i\gamma)^2]+(\omega_{pe}/2n_0)(\omega_{pe}+\omega_k-\omega_0)G_{k\omega}(T_e/T_i)P=0 \tag{7}$$

$$G_{k\omega}=\frac{U_{k\omega}}{\delta n_{k\omega}}=\frac{n_0}{T_i}\left[(R-iY)^{-1}-\frac{T_e}{T_i}\left(1-i\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_e}\right)\right]^{-1} \tag{8}$$

$$R=\sqrt{2}\frac{\omega}{kv_i}\exp\left[-\frac{\omega^2}{2(kv_i)^2}\right]\int\limits_0^{\omega/\sqrt{2}kv_i}\exp t^2dt-1$$

$$Y=\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_i}\exp\left[-\frac{\omega^2}{2(kv_i)^2}\right],\quad P=\frac{E_0^2}{8\pi n_0T_e}\cos^2\widehat{(\mathbf{E}_0\mathbf{k})} \tag{9}$$

where v_e and v_i are the thermal velocities of the electrons and ions. In the derivation of (6) and (7) it was assumed that $|\omega|\ll kv_e$; all the instabilities considered below lie in this region of frequencies.

We note that (7) is applicable only when $P\ll 1$, when the velocity of the electron oscillations in the external field is much less than their thermal velocity. The dispersion equation used in [4] and based on the successive kinetic approach differs from (7) in allowing for electron–electron and electron–ion collisions, the first of which in a majority of cases makes a small contribution to the low-frequency decrement in the attenuation compared with Landau damping, while the second are allowed for phenomenologically in (1). Moreover, small corrections to the electron plasma frequency in terms of the parameter $(m/M)^{1/2}$ are retained in this equation. The simplified dynamic description based on averaging over the fast time ω_{pe}^{-1} thus retains all the features of the kinetics. On the other hand it allows one to examine rather simply the nonlinear stage in the development of an instability.

Instabilities in a nonisothermal plasma have been well studied [4]. For example, the aperiodic ($\mathrm{Re}\,\omega=0$) and decay instabilities differ when $\omega_0-\omega_{pe}\gg\gamma$. The aperiodic instability has a maximum increment

$$\Gamma_{max}=-\gamma+\omega_{pe}T_e(T_e+T_i)^{-1}P/4 \tag{10}$$

at the surface

$$\omega_k+\omega_{pe}-\omega_0=\omega_{pe}T_e(T_e+T_i)^{-1}P/4$$

The minimum threshold is $P_{min}=4(T_e+T_i)T_e^{-1}\gamma/\omega_{pe}$. In a nonisothermal plasma ($T_e\gg T_i$) the condition $kv_i\gg\Gamma$ of applicability of (10) is rapidly violated with an increase in the field E_0. However, Eq. (10) remains valid when $kv_i\ll\Gamma\ll\Omega_k$ (Ω_k is the ion sonic frequency), as follows from (7)-(9). Thus, the nature of the aperiodic instability does not depend on the temperature ratio.

The decay instability in a nonisothermal plasma has a much lower threshold than the aperiodic instability

$$P_{min}=8\frac{\gamma\gamma_s}{\omega_{pe}\Omega_k},\quad \gamma_s=\sqrt{\frac{\pi}{8}}\left(\frac{m}{M}\right)^{1/2}\Omega_k \tag{11}$$

and corresponds to the limiting case $|\omega|\gg kv_i$; γ_s is the low-frequency decrement in attenuation.

The maximum increment is reached at the surface

$$\omega_0-\omega_{pe}=\omega_k+\Omega_k \tag{12}$$

$$\Gamma_{max} = {}^1/_2 [-(\gamma + \gamma_s) + \sqrt{(\gamma - \gamma_s)^2 + {}^1/_2 P \omega_{pe} \Omega_k}\ ] \tag{13}$$

At this surface Re $\omega = \Omega_k$. The region of applicability of (13), as for (10), is limited by fields at which $\Omega_k \gg \Gamma$. With an increase in the ion temperature the surface (12) is found in a region with strong Landau damping on the ions. In this region one must depart from the exact equations (7) and (8) in order to clarify the nature of the instability.

For frequency differences $\omega_0 - \omega_k - \omega_{pe} \gg \gamma$ it follows from (7) that when $\Gamma \ll$ Re ω

$$\omega_0 - \omega_{pe} - \omega_k = \text{Re}\ \omega^* \tag{14}$$

In this case for the threshold value of the field one obtains

$$P = 4\frac{T_i}{T_e}\frac{\gamma}{\omega_{pe}} a\left(\frac{a}{b} + \frac{b}{a}\right), \quad a = \frac{R}{R^2 + Y^2} - \frac{T_e}{T_i},$$
$$b = \frac{Y}{R^2 + Y^2} + \sqrt{\frac{\pi}{2}}\frac{T_e}{T_i}\frac{\omega}{kv_e} \tag{15}$$

The numerical solution of (14) and (15) makes it possible to determine the minimum threshold and the surface of the maximum in the increment in the case of $T_e \sim T_i$. The calculations show that when $T_e \approx 3/4$ T_i the thresholds in the decay and aperiodic instabilities are comparable. Figure 1 shows the dependence of the ratio η of minima of the decay and aperiodic instability thresholds on the temperature ratio. Strong damping on the ions also changes the dependence of the increment on the external field in this region compared with the nonisothermal case. When $T_e \sim T_i$

$$\Gamma_{max} \approx \frac{\omega_{pe}}{4}\frac{T_e}{T_i}\frac{P}{b}, \quad \text{Re}\ \omega \gg \Gamma \gg \gamma \tag{16}$$

The value of b is taken at the surface (14). The dependence of the increment $\tilde{\Gamma} = 4\ \Gamma/\omega_{pe}$ P on the relative frequency difference $\delta = (\omega_k - \omega_0 + \omega_{pe}) / (T_e/M)^{1/2}$ for the values $T_e/T_i = 1$ and 4 is shown in Fig. 2. The condition (14), as it is easy to show, means that $|\psi_+| \ll |\psi_-|$. Thus, in an approximation linear with respect to the disturbances, pairs of waves ($\delta n_{k\omega}$ and ψ_-) with opposite wave vectors are formed and develop for which the sum of the phases, as seen from (8), is a fully determined value. This fact is obviously not connected with the kinetics.

Thus, in an isothermal plasma one must distinguish two instabilities having comparable thresholds, the same dependence of the increments on the external field, and developing in different regions of k-space. Disturbances with wave vectors at the surface $\omega_k = \omega_0 - \omega_{pe}$ which separates the regions indicated above are not excited at all in the framework of (7).

The conclusions presented are necessary for a study of the nonlinear stage of development of an instability.

In conclusion the author thanks V. E. Zakharov for the formulation of the problem and a helpful discussion, as well as A. M. Rubenchik for valuable remarks.

LITERATURE CITED

1. V. N. Oraevskii and R. Z. Sagdeev, "Stability of steady longitudinal oscillations of a plasma," Zh. Tekh. Fiz., 32, No. 11 (1962).
2. V. E. Zakharov, " Collapse of Langmuir forces," Zh. Éksperim. i Teor. Fiz., 60, No. 5 (1971).
3. A. G. Litvak and V. Yu. Trakhtengerts, "Induced scattering of waves and heating of a plasma by coherent radiation," Zh. Éksperim. i Teor. Fiz., 60, No. 5 (1971).
4. N. E. Andreev, A. Yu. Kirii, and V. P. Silin, "Parametric excitation of longitudinal oscillations in a plasma by a weak high-frequency electric field," Zh. Éksperim. i Teor. Fiz., 57, No. 3 (1969).